ROLL No. ____________

H—2016

II SEMESTER EXAMINATION, 2014

M. Sc.

MATHEMATICS

Paper IV

[Advanced Complex Analysis-II]

Time : Three Hours] **[M. M. : 80**

Note : *Attempt any two parts from each question. All questions carry equal marks.*

UNIT – I

1. (a) For $z \neq 0, -1, -2, \ldots\ldots$ prove that

$$\Gamma(z) = \lim_{n \to \infty} \frac{\lfloor n \; n^z}{z(z+1)\ldots\ldots(z+n)}$$

(b) State and prove Mittag-Leffler's theorem.

(c) Let $S = \{z : \operatorname{Re} z \geq a\}$ where $a > 1$. Then for a given $\varepsilon > 0, \exists\, \delta\; 0 < \delta < 1$ such that for all $z \in \delta$ prove that $\left| \int_{\alpha}^{\beta} (e^t - 1)^{-1} t^{z-1}\, dt \right| < \varepsilon$ where $\delta > \beta > \alpha$.

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UNIT – II

2. (a) State and prove Schwarz's Reflexion Principle for symmetric region.

(b) Let $v : [0, 1] \to C$ be a path from a to b and let $\{(f_t, D_t) : 0 \leq t \leq 1\}$ and $\{(g_t, B_t) : 0 \leq t \leq 1\}$ be analytic continuations along v such that $[f_0]_a = [g_0]_a$. Then prove that $[f_1]_b = [g_1]_b$.

(c) Show that when $0 < b < 1$, the series

$$\frac{1}{2} \log (1 + b^2) + i \tan^{-1} b + \frac{z - ib}{1 + ib} - \frac{1}{2} \frac{(z - ib)^2}{(1 + ib)^2} + \ldots\ldots$$

is analytic continuation of the function defined by the series

$$z - \frac{1}{2} z^2 + \frac{1}{3} z^3 - \frac{1}{4} z^4 + \ldots\ldots\ldots$$

UNIT – III

3. (a) Let G be a region and let u be a continuous real valued function on G with the MVP. If there is a point a in G such that $u(a) \geq u(z)$ for all z in G, then prove that u is a constant function.

(b) State and prove Harnack's inequality.

(c) Let $u : G \to \mathfrak{R}$ be a continuous function which has the mean value property. Then prove that u is harmonic.

UNIT – IV

4. (a) State and prove Jensen's formula.

   (b) State and prove Borel's theorem.

   (c) State and prove Hadamard's factorization theorem.

UNIT – V

5. (a) State and prove Schottky's theorem.

   (b) State and prove Montel caratheodory theorem.

   (c) Let f be analytic in $D = \{z : |z| < 1\}$ and let $f(0) = 0$, $f(0) = 1$ and $|f(z)| \leq M$ for all z in D. Then prove that $M \geq 1$ and $f(D) \supset B\left(0\,;\frac{1}{6M}\right)$.

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