ROLL No. _____

H—1017

I Semester Examination, 2013

M. Sc.

MATHEMATICS

Paper V

[Advanced Discrete Mathematics-I]

Time : Three Hours] **[M. M. : 80**

Note : Solve any two parts from each question. All questions are compulsory and carry equal marks.

UNIT – I

1. (a) Write the following sentences into symbols :

(i) The square of any rational number is not 2.

(ii) Two non-parallel coplanar straight lines have a common point.

(iii) If there is no prize, then a person does not purchase a ticket.

P. T. O.

(b) Negate the statement :

(i) "He is poor and laborious."

(ii) "It is daylight and all the people have arisen."

(c) Define semigroup and monoid with example.

UNIT – II

2. (a) Define monoid homomorphism.

(b) Prove that for any commutative monoid $< M, * >$, the set of idempotent elements of M forms a submonoid.

(c) Define subsemigroup and submonoid. Give an example of subsemigroup which is not a submonoid.

UNIT – III

3. (a) Let $(L, \leq)$ be a lattice. Then prove that for any $a, b \in L$.

(i) $a \leq b \Leftrightarrow a \wedge b = a$

(ii) $a \leq b \Leftrightarrow a \vee b = b$

(b) Show that dual of a lattice is a lattice.

(c) In a distributive lattice $(L, \leq)$, if an element has a complement, then prove that this complement is unique.

UNIT – IV

4. (a) Change the following Boolean function to disjunctive normal form :

$$f(x, y, z, t) = [x'y + xyz' + xy'z + x'y'z'y + t']'$$

(b) Prove that every function without constants of Boolean algebra is equal to a function in conjunctive normal form.

(c) Give a short note on Three Variable Karnaugh Map.

UNIT – V

5. (a) If $G = (V_N, \Sigma, P, S)$ is a context free grammar, then prove that $S \Rightarrow \alpha$ if and only if there is a derivation tree in grammar G with yield α.

(b) Prove Kleene's theorem.

(c) Write short note on Polish Notation.

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