Roll No. ....

A–2666

M. A./M. Sc. (Final) EXAMINATION, 2017

MATHEMATICS

(Compulsory)

Paper First

(Integration Theory and Functional Analysis)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove Randon Nikodym theorem.

   (b) Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is measurable rectangle $A \times B$, then show that :

   $$\lambda(A \times B) = \Sigma \lambda(A_i \times B_i)$$

   (c) Let E be a measurable set such that $0 < \nu(E) < \infty$, then prove, that there is a positive set A contained in E with $\nu(A) > 0$.

Unit—II

2. (a) State and prove Riesz-Markoff theorem.

   (b) Explain regularity of measures on locally compact spaces.

(c) If μ is finite Baire measure on the real line, then show that its cumulative distribution function F is monotone increasing bounded function which is continuous on the right, more over $\lim_{n\to\infty} F(X) = 0$.

Unit—III

3. (a) Let $c\,[a, b]$ be a function space of real valued continuous functions defined on $[a, b]$. Define $\|\cdot\|_\infty$ by $\|\cdot\|_\infty : c\,[a,b] \to R$, where $\|f\|_\infty = \max_{t\in[a,b]} |f(t)|$, then prove that $(c[a,b], \|\cdot\|_\infty)$ is Banach space.

(b) If X is a finite dimensional normed space, then prove that strong convergence is equivalent to weak convergence.

(c) Explain basic properties of finite dimensional normed linear spaces and compactness.

Unit—IV

4. (a) Let $T : X \to Y$ be a linear operator. If T is compact then prove that its adjoint operator is also compact.

(b) State and prove Hahn Banach theorem for real linear spaces.

(c) Prove that a Banach space is reflexive if and only if its dual space is reflexive.

Unit—V

5. (a) Prove that every Hilbert space is reflexive.

(b) Prove that product of two bounded self-adjoint operators S and T on a Hilbert space H is self-adjoint iff the operators commute i.e., if ST = TS.

(c) State and prove the generalized Lax-Milgram theorem.

A-77