Roll No. ....

Z–2703

M. A./M. Sc. (Previous) EXAMINATION, 2016

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] [*Maximum Marks : 100*

Note : All questions are compulsory. Solve any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Prove that countable union of countable sets is countable.

   (b) Prove that arbitrary intersection of closed sets in a topological space is a closed set.

   (c) Define a topology in terms of Kuratowski closure operator.

Unit—II

2. (a) Prove that every second countable space is first countable.

   (b) Prove that a mapping f from a topological space X onto another topological space Y is continuous iff $\overline{f^{-1}(B)} \subseteq f^{-1}(\bar{B})$ for every $B \subseteq Y$.

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(c) Prove that a topological space (X, τ) is a T_1 space iff every singleton set $\{x\}$ in X as closed.

Unit—III

3. (a) Prove that every sequentially compact space is countably compact.

(b) Define one point compactification. Let (X, τ) be a non-compact topological space and let $X^* = X \cup \{\infty\}$, where ∞ is an object not belonging to X. Let $\tau^* = \{G^* \subseteq X^* : G^* \in \tau \text{ or } X^* - G^*$ is τ-closed compact subset of $X\}$. Then prove that τ^* is a topology for X^* such that the space (X^*, τ^*) is compact and (X, τ) is a dense subspace of (X^*, τ^*).

(c) Show that the connected components of a topological space are closed.

Unit—IV

4. (a) Let f be a map of a space Y onto a product space $X = \prod_{\alpha \in \wedge} X_\alpha$. Then prove that f is continuous iff the composition map $\pi_i . o f : Y \to X_i$ is continuous.

(b) State and prove embedding lemma.

(c) State and prove Smirnov Metrization theorem.

Unit—V

5. (a) Prove that the relation '$\simeq$' on homotopic paths in a topological space is an equivalence relation.

(b) Let (X, τ) be a topological space and $Y \subseteq X$. Then show that Y is τ-open iff no net in X–Y can converge to a point in Y.

(c) Let T be an ultrafilter on a set X. Then show that $\bigcap\{A : A \in T\}$ is either empty or a singleton subset of X.