Roll No. ....

A–2661

M. A./M. Sc. (Previous) EXAMINATION, 2017

MATHEMATICS

Paper First

(Advanced Abstract Algebra)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) State and prove Jordan-Holder theorem for finite groups.

(b) Prove that, every subgroup of a solvable group is solvable.

(c) If E be an algebraic extension of a field F and $\sigma : F \rightarrow L$ be an embedding of F into an algebraically closed field L. Then prove that σ can be extended to an embedding $\eta : E \rightarrow L$.

2. (a) Prove that, the set Aut (K) of all automorphism of a field K form a group under composition of mapping.

(b) Find the Galois group of $x^3 - 2 \in Q[x]$.

(c) Show that the polynomial $x^5 - 9x + 3$ is not solvable by radicals over Q.

3. (a) Prove that, the necessary and sufficient condition for an R-module M to be a direct sum of its two sub-modules N_1 and N_2 are that :

   (i) $M = N_1 + N_2$

   (ii) $N_1 \cap N_2 = \{0\}$

   (b) If M be a finitely generated free module over a commutative ring R, then prove that all bases of M have the same number of elements.

   (c) State and prove Hilbert basis theorem.

4. (a) Prove that, let $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k \in F$ be distinct characteristic roots of $T \in A(V)$ and let $v_1, v_2, \ldots\ldots, v_k$ be characteristic vectors of T belonging to $\lambda_1, \lambda_2, \ldots\ldots\ldots, \lambda_k$ respectively. Then $v_1, v_2, \ldots\ldots\ldots, v_k$ are linearly independent over F.

   (b) If V be n-dimensional over F and if $T \in A(V)$ has all its characteristics roots in F. Then prove that T satisfies a polynomial of degree n over F.

   (c) Find the Jordan normal form of the matrix :

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -4 & -2 & 0 \\ 4 & 4 & 2 \end{bmatrix}$$

5. (a) Let A be an $m \times n$ matrix over R. If $E_{ij} = 1 - e_{ii} - e_{jj} + e_{ij} + e_{ji}$. Then prove that E_{ij} A is the matrix obtained from A by interchanging the ith and the jth rows and also prove that :

$$E_{ij}^{-1} = E_{ij}$$

(b) Find the invariant factors of the matrix :

$$\begin{bmatrix} -x & 4 & -2 \\ -3 & 8-x & 3 \\ 4 & -8 & -2-x \end{bmatrix}$$

over the ring Q [x]. Also find the rank.

(c) Reduce the following matrix A to a rational canonical form :

$$A = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$

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