Roll No. ....

E–3825

M. Sc./M. A. (Previous) EXAMINATION, 2021

MATHEMATICS

Paper Fifth

(Advance Discrete Mathematics)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) Define quantifier and predicate with examples.

   (b) What do you mean by valid ? Test the validity of the following argument :

   "If it rains then it will be cold. If it is cold then I shall stay at home. Since it rains therefore I shall stay at home."

   (c) Define semigroup homomorphism and monoid homomorphism. Let (S, *) be a semigroup and R be a congruence relation on (S, *). The quotient set S/R is a semigroup (S/R, $\oplus$), where the operation $\oplus$ corresponds to the operation * on S. Also, there exists a homomorphism from (S, *) onto (S/R, $\oplus$) called the natural homomorphism.

2. (a) Let $(L, \leq)$ be a lattice. Then the following results hold :

   (i) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$

   (ii) $a \vee (a \wedge b) = a$

   (b) Two bounded lattices L_1 and L_2 are complemented iff $L_1 \times L_2$ is complemented.

   (c) Replace the switching function :

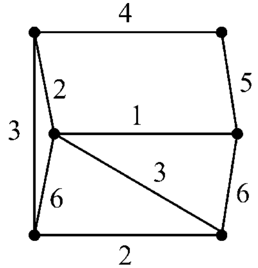
$$F(x, y, z) = x \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$$

   by a simpler switching circuit and verify the equivalent circuits by truth tables.

3. (a) Let G be a simple graph with n vertices. If G has k components, then the maximum number of edges that G can have is $\dfrac{(n-k)(n-k+1)}{2}$.

   (b) The necessary and sufficient condition for a connected graph G to be a Euler graph is that 'all vertices of G are of even degree'.

   (c) State the Kruskal's algorithm. Find the minimal spanning tree for the graph in figure using both Kruskal and Prim's method :



4. (a) Find π_0, π_1 and π_2 for the following finite state machine :

State	Input		Output
	0	1	
⇒ S_0	S_1	S_5	0
S_1	S_0	S_5	0
S_2	S_6	S_0	0
S_3	S_7	S_1	0
S_4	S_0	S_6	0
S_5	S_7	S_2	1
S_6	S_0	S_3	1
S_7	S_0	S_2	1

(b) Describe Moore and Mealy machines with examples.

(c) Show that the two finite state machines shown in the following tables are equivalent :

(i)

State	Input		Output
	1	2	
⇒ A	B	C	0
B	B	D	0
C	A	E	0
D	B	E	0
E	F	E	0
F	A	D	1
G	B	C	1

(ii)

State	Input		Output
	1	2	
⇒ A	H	C	0
B	G	B	0
C	A	B	0
D	D	C	0
E	H	B	0
F	D	E	1
G	H	C	1
H	A	E	0

5. (a) State and prove Pumping Lemma.

(b) Construct a grammar for the language $L = \{a^x b^y : x > y > 0\}$.

(c) Consider the context free grammar G, that consists of the following productions :

$S \rightarrow aB/bA$

$A \rightarrow a/aS/bAA$

$B \rightarrow b/bS/aBB$

For the string *aa bb ab ab*, find :

(i) Leftmost derivation

(ii) Rightmost derivation

(iii) Parse tree