Roll No. ....

Z–312

M. A./M. Sc. (First Semester) EXAMINATION, Dec., 2015

MATHEMATICS

Paper Fourth

(Complex Analysis—I)

Time : Three Hours] [*Maximum Marks : 80*

[*Minimum Pass Marks : 16*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove Cauchy's Integral formula.

(b) Prove that :

$$\log(1-z) = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} + \ldots\ldots + (-1)^n \frac{(z-1)^n}{n} + \ldots\ldots$$

(c) State and prove Liouville's theorem.

Unit—II

2. (a) Every polynomial of degree n has n zeros.

(b) Use Rouche's theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$.

(c) State and prove Schwarz lemma.

Unit—III

3. (a) Show that :

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}},\ a > b > 0$$

(b) Apply the calculus of residue, to prove that :

$$\int_0^{\infty} \frac{dx}{1 + x^2} = \frac{\pi}{2}$$

(c) Use the method of contour integration to prove that :

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

Unit—IV

4. (a) To determine the bilinear transformation which transform three distinct points into three specified distinct points.

(b) To find all the bilinear transformation which maps the half plane $I(z) \geq 0$ onto the unit circular disc $|w| \leq 1$.

(c) Discuss the application of the transformation $w = z^2$ to the area in the first quadrant of the z-plane bounded by the axes and circles :

$$|z| = a,\ |z| = b,\ (a > b > 0).$$

Unit—V

5. (a) State and prove Weierstrass's theorem.

   (b) State and prove Hurwitz's theorem.

   (c) State and prove open mapping theorem.

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