Roll No. ....

Z–311

M. A./M. Sc. (First Semester)
EXAMINATION, Dec., 2015

MATHEMATICS

Paper Third

(Topology—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) (i) Write short notes on the following :

(A) Countable set

(B) Cardinal number

(C) Prove that $N \times N$ is countable.

(b) (i) If (X, T) is a topological space and $A, B \subset X$, then prove that :

(A) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$

(B) $A \subset B \Rightarrow A^0 \subset B^0$, where A^0 denotes the interior of A.

(C) If (X, T) is a topological space and $A \subset X$, then prove that A is open if and only if it is a neighbourhood of each of its points.

(c) Define derived set and dense set with an example. Show that a subset A of a topological spaces X is dense in X if and only if for every non-empty open subset B of X, $A \cap B \neq \phi$.

Unit—II

2. (a) Define the following with an example :

   (i) Continuous function

   (ii) Homeomorphism

   (iii) Separable space

   (iv) Lindeloff space

   (b) (i) If a topological space (X, T) is second countable, then prove that every open cover of it has a countable subcover.

   (ii) Show that compositions of continuous functions are continuous.

   (c) Define first countable space and show that first countability is a hereditary property.

Unit—III

3. (a) Define the following with an example :

   (i) T_1-space

   (ii) Regular space

   (iii) Normal sapce

   (iv) Completely regular space

   (b) (i) Prove that every subspace of a T_2-space is a T_2-space.

(ii) If X is a regular space, then prove that for any $x \in X$ and any open set G containing x there exists an open set H containing x such that $\bar{H} \subset G$.

(c) (i) Show that every T_4-space is a T_3-space.

(ii) If X is a normal space, then prove that for any closed set C and any open set G containing C, there exists an open set H and a closed set K such that $C \subset H \subset K \subset G$.

Unit—IV

4. (a) (i) Show that every map from a compact space into a Hausdorff is closed.

(ii) Define the following with an example :

(A) Finite intersection property

(B) Compact space

(b) (i) Prove that a closed subspace of a compact space is compact.

(ii) Show that a continuous image of a sequentially compact set is sequentially compact.

(c) Define locally compact space and prove that every open subspace of a locally compact space is locally compact.

Unit—V

5. (a) Define countably compact space and prove that every countably compact metric space is second countable.

(b) Define connected space with an example and prove that a subset of R is connected if and only if it is an interval.

(c) (i) Define the following with an example :

(A) Separated sets

(B) Component

(ii) Define locally connected space. If E be a component in a locally connected space X, then prove that E is open in X.