Paper 1 : CALCULUS & GEOMETRY-2016

Time : Three Hours] *[Maximum Marks : 50*

Note : All questions are compulsory. Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Let $f : [a, b] \to R$ be a bounded function and P is any partition of [a, b], then prove that :

$$L(p, f) \leq U(p, f).$$

(b) Let $f : [a, b] \to R$ be a bounded function. Then show that for every $\in > 0$ there corresponds $\delta > 0$ such that :

$$U(p, f) < \int_a^{-b} f(x)\,dx + \in \quad \text{and} \quad L(p, f) > \int_{-a}^{b} f(x)\,dx - \in.$$

(c) Let $f \in R$ [a, b] and let F be a differentiable function on [a, b] such that $F'(x) = f(x)$ for all $x \in [a, b]$. Then show that :

$$\int_a^b f(x)\,dx = F(b) - f(a)$$

UNIT - 2

2. (a) Discuss the maximum and minimum of the function :

$$f(x, y) = x^2 + y^2 + 6x + 12$$

(b) In any triangle ABC, find the maximum value of cos A cos B cos C by Lagrange's method.

(c) Find the minimum value of $u = x^2 + y^2 + z^2$ having given $ax + by + cz = p$.

UNIT - 3

3. (a) To test the convergence of integral $\int_a^{\infty} \frac{dx}{x^n}$, where $a > 0$.

(b) Test for convergence of the integral :

$$\int_0^{\infty} \frac{dx}{\frac{1}{x^3}\left(1+x^{\frac{1}{2}}\right)}$$

(c) Test the convergence of the integral $\int_a^{\infty} \frac{\sin mx}{a^2+x^2} dx$

UNIT - 4

4. (a) Find the equation of the cone whose vertex is (a, b, c) and base curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0.$

(b) Prove that the equation of the right circular cone whose vertex is the origin, axis is z-axis and semi vertical angle is α is $x^2 + y^2 = z^2 \tan^2 \alpha$.

(c) Find the equation of right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.

UNIT - 5

5. (a) Explain the relation between Cartesian and Polar co-ordinates.

(b) In an ellipse $\frac{2}{r}=1-e\cos\theta$ if PQ is a chord passing through focus S then prove that $\frac{1}{SP}+\frac{1}{SQ}=1.$

(c) To find the polar equation of a conic with its latus rectum of length 2 *l*, eccentricity e and the focus being pole.