Paper 1 : CALCULUS & GEOMETRY-2012

Note : Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Let $f : [a, b] \rightarrow R$ is a bounded function in [a, b]. Then f is R-integrable if and only if for each $\in > 0$ there is a partition P such that : $U(P, f) - L(P, f) < \in$

   (b) If $f : [a, b] \rightarrow R$ is a contitnuous function, then f is R-integrable in [a, b].

   (c) Function f is defined on interval [0, 1] as $f(x) = x \,\forall\, x \in [0,$

   1]. Divided [0, 1] into n equal parts and find $\underline{\int_0^1} x\,dx, \overline{\int_0^1} x\,dx$,

   show that : $f \in R[0, 1]$ and $\int_0^1 x\,dx = \frac{1}{2}$.

UNIT - 2

2. (a) Find maximum or minimum value of the function

$$x^3y^2(1 - x - y).$$

   (b) Find out the point inside the triangle, whose sum of the squares of the distances from vertices is minimum.

   (c) Find minimum or maximum value of the function $u = x^2 + y^2 + z^2$ when $ax^2 + by^2 + cz^2 = 1$.

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UNIT - 3

3. (a) Test for the convergent of the integral : $\int_a^\infty e^{-x} \frac{\sin x}{x^2} dx, a > 0$

   (b) Prove that by using μ test $\int_1^0 \frac{dx}{\sqrt{x(1-x)}}$ is convergent.

   (c) Show that $\int_9^{\pi/2} \log \sin x \, dx$ is convergent.

UNIT - 4

4. (a) Show that the following equation represents a cone and find co-ordinate of vertex :

$$4x^2 - y^2 + 2z^2 + 2xy - 3yz - 12x - 11y + 6z + 4 = 0$$

   (b) Find out the equation of right circular cone, whose vertex is origin and semivertical angle is 45°.

   (c) Find the equation of cylinder, whose generator is parallel to line $x = -\frac{y}{2} = \frac{z}{3}$ and co-ordinate curve is the ellipse $x^2 + 2y^2 = 1, z = 3.$

UNIT - 5

5. (a) If PSP' is a focal cord of the conic $\frac{l}{r} = 1 + e\cos\theta$ where S be focus, then show that : $\frac{1}{SP} + \frac{1}{SP'} = \frac{2}{l}$

(b) Find the equation of the circle whose centre is at the point (4, 5) and which passes through the centre of the circle :

$$x^2 + y^2 + 4y - 6x - 12 = 0$$

(c) In the conic $\frac{l}{r} = 1 + e\cos\theta$, there is two points, which are the direction angles α and β and these points are the end points f diameter. Prove that : $\tan\frac{\alpha}{2}.\tan\frac{\beta}{2} = \frac{e+1}{e-1}$