Paper 2 : DIFFERENTIATION & INTEGRATION-2012

UNIT - 1

1. (a) If : $\frac{1}{ym} + y - \frac{1}{m} = 2x$ then prove that :

$$(x^2 - 1)\, y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 - m^2)\, y_n = 0$$

(b) For the following pair of functions find the value of 'r' of the Cauchy's men value theorem in the interval [a , b] :

$$f(x) = e^x,\ 4(x) = e^{-x}$$

(c) Using Taylor's theorem prove that :

$$\tan^{-1}(x+h) = \tan^{-1} x + h \sin z \frac{\sin z}{1}$$

$$-(h \sin z)^2 \frac{\sin 2z}{2} + (h \sin z)^3 \frac{\sin 3z}{3} + \dots\dots\dots$$

$$+(-1)^{n-1}(h \sin z)^n \frac{\sin nz}{n} + \dots\dots\dots$$

where $z = \cot^{-1} x$.

UNIT - 2

2. (a) Find the asymptotes of the following curve :

$$y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x + 1 = 0$$

(b) For the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

prove that the radius of curvature at an end of the major axis is equal to the semi latus-rectum of the ellipse.

(c) Find the points of inflexion on the curve : $x = \log (y/x)$

UNIT - 3

3. (a) If $u = f(r)$, where $r^2 = x^2 + y^2$, show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(b) If : $u = \sin^{-1} \frac{x + y}{\sqrt{x} + \sqrt{y}}$

then prove that : $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

(c) If :

$$u = a \cosh x \cos y$$
$$v = a \sinh x \sin y$$

then show that : $\frac{\partial(u, v)}{\partial(x, y)} = \frac{1}{2} a^2(\cosh 2x - \cos 2y)$

UNIT - 4

4. (a) Find the value of : $\int \frac{dx}{a^2 - b^2 \cos^2 x}$

(b) Evaluate : $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}}$

(c) Prove that : $\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta = \frac{\pi}{8}\log 2$

UNIT - 5

5. (a) Evaluate : $\int_0^2 \int_0^x \int_0^{x+y} e^x (y+2z)\, dx\, dy\, dz$

(b) Change the order of integration in :

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dx\, dy$$

and hence evaluate it.

(c) Find the complete area of the astroid :

$$x^{2/3} + y^{2/3} = a^{2/3}$$