Paper 2 : DIFFERENTIATION & INTEGRATION-2011

Note : Attempt any two parts from each questions. All questions carry equal marks.

UNIT - 1

1. (a) If $u = \sin nx + \cos nx$, then prove that :

$$u_r = n^r [1 + (-1)^r \sin 2nx]^{1/2}$$

(b) State and prove Lagrange's mean value theorem.

(c) Prove that :

$$\sin ax = ax - \frac{a^3x^3}{3!} + \frac{a^5x^5}{5!} - \ldots\ldots + \frac{a^{n-1}x^{n-1}}{(n-1)!}$$

$$\sin\left(\frac{n-1}{2}\pi\right) + \frac{a^nx^n}{n!}\sin\left(a\theta x + \frac{n\pi}{2}\right)$$

where $0 < \theta < 1$.

UNIT - 2

2. (a) Find all asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$$

(b) Find the radius of curvature at the point of the curve $\sqrt{x} + \sqrt{y} + \sqrt{a}$, where line $y = x$ crosses the curve.

(c) Trace the curve : $y^2(2a - x) = x^3$.

UNIT - 3

3. (a) If u is homogeneous function of x and y of degree n, then prove that : $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

(b) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point P (1, 2,3) in the direction PQ, where the co-ordinates of Q is (5, 0, 4).

(c) If $x + y + z = u$; $y + z = uv$, $z = uvw$, then prove that :

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2v$$

UNIT - 4

4. (a) Evaluate : $\int \sec x \log(\sec x + \tan x)\, dx$

(b) Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(2 + \sin x)}\, dx$

(c) Evaluate : $\int_0^{\pi/2} \frac{dx}{4 + 5\cos x}$

UNIT - 5

5. (a) Evaluate : $\int_0^3 \int_1^2 xy(1 + x + y)\, dx\, dy$

(b) Change the order of integration in : $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx\, dy$ and hence evaluate it.

(c) Find area of the region bounded by the curve $y^2 (2a - x) = x^3$ and its asymptote.