Paper 2 : DIFFERENTIATION & INTEGRATION-2009

Note : Attempt any two parts from each question. All questions carry equal marks.

1. (a) What is successive differentiation ? State and explain Leibnitz theorem.
   (b) State and prove Lagrange's mean value theorem.
   (c) Expand $\tan^{-1}x$ in the powers of $(x - \pi/4)$ by Taylor theorem.
2. (a) Find the asympotes of the curve :
   $$x^3 + 3x^2 y - 4x^3 - x + y + 3 = 0$$
   (b) Trace the curve : $y^2(2a - x) = x^3$
   (c) Explain with examples :
   (i) Asymptoes
   (ii) Tracing of Curves
3. (a) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then show that :

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   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

   (b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ the direction of the vector $2i - j - 2k$ at the point $(1, -2, -1)$.
   (c) Explain Jacobian theorem with example.
4. (a) Explain the principle of Integration by parts with example.
   (b) Evaluate : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$
   (c) Evaluate : (i) $\int \frac{\log x}{(1 + \log x)^2}\, dx$ (ii) $\int \sin^7 x\, dx$
5. (a) Change the order of integration and evaluate the following integral :
   $$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{a-x}} dx\, dy$$
   (b) Find area of the curve : $a^2 x^2 = y^2 (2a - x)$
   (c) Evaluate : $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\, dy}{1 + x^2 + y^2}$