DISCRETE MATHEMATICS - 2014

Note : Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Prove that each of the following statements is a tautology :

$$(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$$

(b) Explain the following :
(i) Universal Quantifier (ii) Existential Quantifier

(c) Write the following predicate into symbolic language :
(i) For every real number there is a greater real number.
(ii) Every irrational number is a real number.
(iii) The number divisible by an even number is even.
(iv) Every teacher of a college in learned.
(v) All students are not wise

UNIT - 2

2. (a) Let B be the set of all positive divisors of 30 i.e. :

$$B = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

and the operations $\vee$ and $\wedge$ on B are defined as follows :

$a \vee b$ = L C M of a and b

$a \wedge b$ = H C F of a and b

Prove that $(B, \vee, \wedge)$ is a Boolean Algebra.

(b) To prove that, for any two elements a, b of a Boolean algera:
a + b = least upper bound of a and b
i,e., a + b = lub {a, b}.

(c) Draw the logic circuit with inputs a,b,c and output y which corresponds to the Boolean expression :

$$Y = ab'c + abc' + ab'c'$$

UNIT - 3

3. (a) Show that the number of minimal Boolean function in n-variables are 2^n.

(b) Express the following functions into disjunctive normal form :

$$f(x, y, z) = (x + y + z) \cdot (xy + x' \cdot z)'$$

(c) Design a tree-net in three variables for the flow function :

$$x \cdot y \cdot z + x' \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$$

UNIT - 4

4. (a) If A,B,C are any three non-empty sets, then prove that :

$$(A - B) \times C = (A \times C) - (B \times C)$$

(b) Show that the set Q of rational numbers is countable.

(c) Show that the mapping $f : R_+ \to R$ defined by $f(x) = \log x$, $x \in R_+$ is one-one onto where R_+ is the set of positive real numbers and R is the set of real numbers.

UNIT - 5

5. (a) Explain the basic concept of Graph theory.

(b) Write short notes on the following :
(i) Binary trees (ii) Spanning trees

(c) Write short notes on the following:
(i) Euler circuit (ii) Hamiltonian graph