ROLL No. ___________

H—1016

I Semester Examination, 2013

M. Sc.

MATHEMATICS

Paper IV

[Complex Analysis-I]

Time : Three Hours] **[M. M. : 80**

Note : *Attempt any two questions from each unit. All questions carry equal marks.*

UNIT – I

1. (a) State and prove Liouville's theorem.

(b) If C is closed contour containing the origin inside it, then show that

$$\frac{a^n}{|\underline{n}} = \frac{1}{2\pi i}\int_C \frac{e^{az}}{z^{n+1}}\,dz$$

(c) Stae and prove the Taylor's theorem.

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UNIT – II

2. (a) State and prove maximum modulus principle.

(b) If $a > e$, use Rouche's theorem to prove that the equation $e^z = az^n$ has n roots inside the circle $|z| = 1$.

(c) State and prove Inverse function theorem.

UNIT – III

3. (a) Define residue with one example and find the residue of

$$f(z) = \frac{e^z}{z\,(\sin mz)} \text{ at the origin.}$$

(b) Evaluate by calculus of residues :

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\,dx$$

(c) Evaluate by calculus of residues :

$$\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta}\,d\theta$$

UNIT – IV

4. (a) If a mapping $w = f(z)$ is conformal, then show that $f(z)$ is an analytic function of z.

(b) Prove that every bilinear transformation maps circles or straight lines into circles or straight lines.

(c) Show that the transformation $w = \frac{iz + 2}{4z + i}$ maps the real axis in the z-plane into a circle in the w-plane.

UNIT – V

5. (a) State and prove Riemann mapping theorem.

(b) State and prove Hurwitz's theorem.

(c) Let $\{f_n\}$ be a sequence in H(G) and $f \in$ C (G, C) such that $f_n \to f$. Then prove that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$.

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