Dip H

Roll No.______________ Code No. : H/Sem/1015

# M.Sc. MATHEMATICS
# FIRST SEMESTER EXAMINATION 2013

## Paper – III
## TOPOLOGY

Time - 3 Hrs Maximum Marks – 80

Note : Attempt *any two* parts from each unit. Each question carries equal marks.

### UNIT – I

1. a) Let $\mathcal{B}$ be a base for a topology T on a set X and let $Y \subset X$.
   $Let\, \mathcal{B}/Y = \{B \cap Y : B \in \mathcal{B}\}$ . Then prove that, $\mathcal{B}/Y$ is a base for the topology $T/Y$ $on\, Y$ .
   b) Prove that Metrisability is a hereditary property.
   c) State and prove well ordering theorem.

### UNIT – II

2. a) Let X and Y be topological spaces. Prove that, a mapping $f : X \to Y$ is continuous if and only if $f(\bar{A}) \subset \overline{f(A)}$ for every subset A of X.
   b) Let X be any set and let C be the Kuratowski closure operator on X , i.e. $C : P(X) \to P(X)$ satisfies the axioms of closure operators, then prove that there exists a unique topology T on X such that for each $A \subset X$, C(A) coincides with closure of A.
   c) Prove that every second countable space is Separable.

### UNIT – III

3. a) Prove that Hausdorff property is hereditary.
   b) Define Topological property. Prove that Normality is a topological property.
   c) State and prove Tietze Extension theorem.

## UNIT – IV

4. a) Prove that every Tychonoff space is $T_3$ space.

   b) Prove that the one point compactification of a space is Hausdorff if and only if the space is locally compact and Hausdorff.

   c) Prove that a one-one continuous mapping of a compact space onto a Hausdorff space is a Homeomorphism.

## UNIT – V

5. a) Let $\{C_\lambda : \lambda \in A\}$ be a family of connected subsets of a space X such that $\cap \{C_\lambda : \lambda \in A\} \neq \phi$ . Then prove that $\cup \{C_\lambda : \lambda \in A\}$ is a connected set.

   b) Prove that continuous image of a connected space is connected.

   c) Prove that a metric space is sequentially compact if and only if it has the Bolzano Weirstrass property.

* * *