Roll No. .....

# B–314

## M. A./M. Sc. (First Semester)
## EXAMINATION, Dec., 2017

MATHEMATICS

Paper Fifth

**(Advanced Discrete Mathematics–I)**

*Time : Three Hours ]* *[ Maximum Marks : 80*

*[ Minimum Pass Marks : 16*

**Note :** Attempt all Sections as directed.

**Section—A** 1 each

**(Objective/Multiple Choice Questions)**

**Note :** Attempt all questions. Choose the correct answer.

1. Which of the following is not a statement ?
   (a) Blood is yellow
   (b) $3 + 5 = 8$
   (c) $3 - x = 7$
   (d) 8 is less than 7

2. Consider the statement $p \rightarrow q$. Now which of the following about above statement is incorrect ?
   (a) Its inverse is $\sim p \rightarrow \sim q$
   (b) Its contrapositive is $\sim q \rightarrow \sim p$
   (c) Its converse is $q \rightarrow p$
   (d) None of the above is correct

3. Which of the following is not a semigroup ?

   (a) $(R, *)$ where $a * b = |a - b| \ \forall a, b \in R$

   (b) $(R, +)$

   (c) $(N, .)$

   (d) $(N, +)$

4. Which of the following is correct ?

   (a) Every cyclic monoid is a commutative monoid.

   (b) The identity element in any monoid need not be unique.

   (c) Union of two submonoids of a monoid is again a submonoid.

   (d) All of the above are correct

5. If $f$ is a homomorphism from a semigroup $(S, *)$ to a semigroup $(T, 0)$ and $a \in S$, then which of the following is correct ?

   (a) $a$ is idempotent.

   (b) $a$ is idempotent $\Rightarrow f(a)$ is idempotent in T.

   (c) $a$ is idempotent $\Rightarrow$ each element of T is idempotent.

   (d) $b \in T$ is idempotent $\Rightarrow f^{-1}(b)$ is idempotent in S.

6. If $(N, +)$ and $(\{0, 1, 2, 3\}, +_4)$ are semigroups, then which of the following is correct ?

   (a) $(N, +)$ is isomorphic to $(\{0, 1, 2, 3\}, +_4)$

   (b) $f : N \to \{0, 1, 2, 3\}$ defined by $f(a) = a \pmod 4$ $\forall\ a \in N$ is a semigroup homomorphism.

   (c) There exists no semigroup homomorphism from N to $\{0, 1, 2, 3\}$.

   (d) $(\{0, 1, 2, 3\}, +_4)$ is not a semigroup.

7. $a \equiv b \pmod{m}$ is a congruence relation. If $a = 9$ and $m = 5$, then $b =$

   (a) 1.8

   (b) 14

   (c) 4

   (d) 0

8. If (S, *) and (T, 0) are two monoids and $e$ and $e'$ are their identities respectively, then the identity of the monoid $(S \times T, \oplus)$ is :

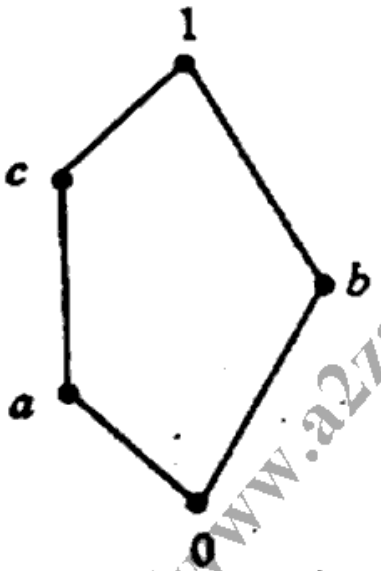
   (a) $e \times e'$

   (b) $(e, e')$

   (c) $(e', e)$

   (d) $e \oplus e'$

9. In the lattice




the atoms are :

   (a) $a$ and $b$

   (b) $c$

   (c) $b$ and $c$

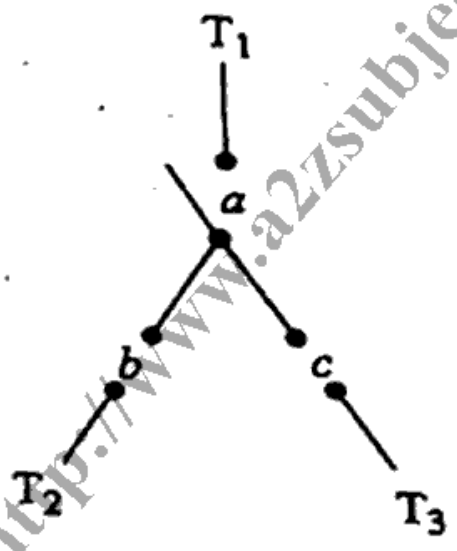
   (d) None of these

10. Which of the following is incorrect ?
    (a) A sublattice of a modular lattice is modular.
    (b) The dual of modular lattice is modular.
    (c) Every lattice having four or less element is always modular.
    (d) None of the above is correct

11. Which of the following is known as the idempotent law in a Boolean algebra (B, +, ., ') ?
    (a) $a + 1 = 1$ and $a.\ 0 = 0\ \forall a \in B$
    (b) $a + a = a$ and $a.\ a = a\ \forall a \in B$
    (c) $a + (a.\ b) = a$ and $a.\ (a + b) = a\ \forall\ a, b \in B$
    (d) $(a+b)' = a'.b'$

12. A Boolean function in 4 variables $x_1$, $x_2$, $x_3$, $x_4$ there are exactly how many minterms ?
    (a) 32
    (b) 16
    (c) 8
    (d) 4

13. The following 3-terminal circuit




is called a :
    (a) Wye circuit

(b) Delta circuit
(c) Star circuit
(d) Bridge circuit

14. Which of the following circuit represents AND gate ?

(a) 

(b) 

(c) 

(d) 

15. The Karnaugh map of $f(x, y) = xy + x'y'$ is :

(a)

| | $x$ | $x'$ |
|---|---|---|
| $y$ | 1 | 1 |
| $y'$ | 0 | 0 |

(b)

| | $x$ | $x'$ |
|---|---|---|
| $y$ | 1 | 0 |
| $y'$ | 0 | 1 |

(c)

| | $x$ | $x'$ |
|---|---|---|
| $y$ | 1 | 0 |
| $y'$ | 1 | 0 |

(d)

| | $x$ | $x'$ |
|---|---|---|
| $y$ | 1 | 1 |
| $y'$ | 1 | 1 |

16. Complement of the Boolean function $f = a'b + ab'$ is :

(a) $ab' + a'b$

(b) $(a + b').(a' + b)$

(c) $ab + a'b'$

(d) $a'b$

17. A language is said to be a type-2 language if it can be specified by a :

(a) type-2 grammar but not by type-3 grammar

(b) type-2 grammar but not by type-1 grammar

(c) type-2 grammar and type-3 grammar

(d) None of the above

18. If A = {0, 1}, then which of the following expressions are regular over A ?

(a) 0 * (0 + 1)

(b) 00* (0 + 1) * 1

(c) (01) * (01 + 1)

(d) All of the above

19. If R = $a + b^*$ is a regular expression over $\{a, b\}$, then L(R) =

(a) $\{a, \lambda, b\}$

(b) $\{a, \lambda, b^2\}$

(c) $\{a, \lambda, b, b^2 .....\}$

(d) None of these

20. Which of the following is not a language over the alphabet $A = \{a, b, c\}$ ?

(a) $\{a^2 c b^2\}$

(b) $\{a^3 b c^2\}$

(c) $\{a, b, ab, aa, ac, abc, cab\}$

(d) $\{a^3 b c^3\}$

## Section—B

$1\frac{1}{2}$ each

### (Very Short Answer Type Questions)

**Note :** Attempt all questions. Answer in 2-3 sentences.

1. What are quantifiers ?
2. Define monoids.
3. Define subsemigroup.
4. State basic homomorphism theorem.
5. Define sublattice.
6. Give example of a switching algebra.
7. Define join-irreducible elements.
8. Define Karnaugh map.
9. Define grammar.
10. State Kleen's theorem.

## Section—C

$2\frac{1}{2}$ each

### (Short Answer Type Questions)

**Note :** Attempt all questions. Answer in less than 75 words.

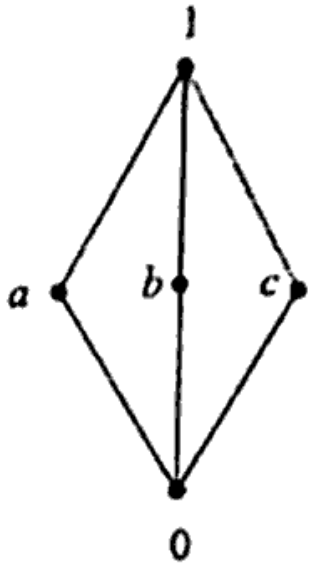
1. Construct the truth-table for $\sim(p \wedge \sim q)$.
2. Give example of a semigroup.
3. Show that the equivalence relation R defined by :

$$aRb \text{ iff } a \equiv b \pmod 2$$

on the semigroup (I, +) of integers under addition is a congruence relation on (I, +).

4. Let R be a congruence relation on the monoid (M, •). Then prove that $\left(\frac{M}{R}, \oplus\right)$ is a monoid where $\oplus$ on $\frac{M}{R}$ is defined by $[a] \oplus [b] = [a \bullet b]$.

5. Examine whether the following lattice is distributive or not :




6. In any Boolean algebra B, show that :

$$a \leq b \Rightarrow a + bc = b(a + c)$$

where $a, b, c \in B$.

7. Find the Boolean function of three variables $x$, $y$ and $z$ which is 1 if either $x = y = 1$ and $z = 0$ or if $x = 1 = z$ and $y = 0$ and is zero otherwise.

8. Construct a circuit using gates to realize the Boolean expression :

$$f = (x_1 + x_2)(x_1' + x_3) + (x_3 + x_4)'$$

9. Construct a grammar for the language :

$$L = \{a^i b^{2i} : i \geq 1\}$$

10. Explain Polish Notations.

## Section—D

4 each

**(Long Answer Type Questions)**

**Note :** Attempt all questions. Answer using less than 150 words for each.

1. Show that the following argument is valid :

$$\frac{\begin{array}{c} p \\ p \to q \\ q \to r \end{array}}{r}$$

*Or*

Prove that every cyclic monoid is a commutative monoid.

2. Let (S, *) and (T, 0) be two monoids with identities $e$ and $e'$ respectively. Prove that $(S \times T, \oplus)$ is a monoid with identity $(e, e')$ where $\oplus$ is defined by :

$$(a, b) \oplus (c, d) = (a * c, b \circ d) \text{ for all } (a, b)(c, d) \in S \times T$$

*Or*

Show that the monoids $(\{0, 1, 2, 3\}, +_4)$ and $(\{1, 3, 7, 9\}, \times_{10})$ are isomorphic.

3. Prove that every chain is a distributive lattice.

*Or*

Prove that no Boolean algebra can have three distinct elements.

4. Write the function $(xy' + xz)' + x'$ in conjunctive normal form.

*Or*

Use a Karnaugh map to find a minimal form of the function :

$$f(x, y, z, w) = x\,y\,z\,w + x\,y\,z\,w' + x\,y'\,z\,w' + x'y'z\,w + x'y'\,z\,w'$$

5. Find the language L(G) over $\{a, b\}$ generated by the grammar $G = (\{a, b\}, \{S, C\}, S, P)$ where P consists of $S \rightarrow aCa$, $C \rightarrow aCa$ and $C \rightarrow b$.

*Or*

State and prove pumping lemma.



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