Roll No. ....

X–294

M. A./M. Sc. (First Semester) EXAMINATION, Dec., 2013

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Solve any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define Tautology and show that :

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$$

(b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

(c) What are the Quantifiers ? Explain universal quantifier and existential quantifier.

Unit—II

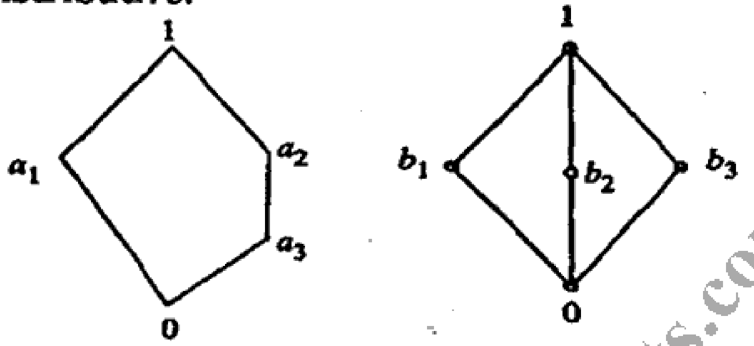
2. (a) Define submonoid and prove that for any commutative monoid $(M, *)$, the set of idempotent elements of M forms a submonoid.

(b) Define semigroup homomorphism and let X be a set containing n elements, let X^* denote free semigroup generated by X and let (S,⊕) by any other semigroup generated by any n generators, then show that there exists a homomorphism $g : X^* \to S$.

(c) Let (S, *) be a given semigroup, then show that there exists a homomorphism $g : S \to S^S$, where $(S^S, 0)$ is a semigroup of functions from S to S under the operation of (left) composition.

Unit—III

3. (a) Define distributive lattice and show that the lattices given by the following diagrams in fig. (a) are not distributive.



(b) Prove the De Morgan's law.

(c) Define the following with example :

(i) Sublattice

(ii) Direct products

(iii) Boolean homomorphism

(iv) Complete lattice

Unit—IV

4. (a) Write the following Boolean expressions in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3 :

(i) $x_1 * x_2$

(ii) $x_1 \oplus x_2$

(b) Use the Karnaugh map representation to find a minimal sum-of-product of the following function :

$$f = \Sigma\,(10, 12, 13, 14, 15)$$

(c) Define gates and draw the logical expression with inputs *a*, *b* and output *f* where :

$$f = (a + b) \cdot (\bar{a} + \bar{b})$$

Unit—V

5. (a) State and prove Pumping lemma.

(b) Define grammar and the language $L(G_3) = \{a^n\, b^n\, c^n \mid n \geq 1\}$ is generated by the following grammar :

$$G_3 = \langle \{S, B, C\}, \{a, b, c\}, S\ \phi \rangle$$

where ϕ consists of the productions

$S \rightarrow a\,SBC$

$S \rightarrow aBC$

$CB \rightarrow BC$

$aB \rightarrow ab$

$bB \rightarrow bb$

$bC \rightarrow bc$

$cC \rightarrow cc$

Then find the derivation for the string $a^2\, b^2\, c^2$.

(c) Define the following :

(i) Rewriting rules

(ii) Language generated by a grammar

(iii) Context-free grammar

(iv) Conversion of infix expressions

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