Roll No. ....

X–292

M. A./M. Sc. (First Semester) EXAMINATION, Dec., 2013

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt *one* question from each Unit compulsorily. All questions carry equal marks.

Unit—I

1. (a) Define cardinal number with an example. Prove that the countable union of countable sets is countable.

   (b) Find the closure of the set $\left\{2, \frac{3}{2}, \frac{4}{5}, \frac{5}{4}, \frac{6}{5}, \ldots\ldots\ldots\right\}$. Prove that a subset A of a topological space is closed if and only if $\overline{A} = A$.

2. (a) Show that a subspace of a topological space is itself a topological space.

   (b) Define Cantor set. State and prove Cantor's theorem.

Unit—II

3. (a) Define Kuratowski space. Prove that every second countable is first countable.

(b) Give an example of continuous function. Prove that Homeomorphism is an equivalence relation in the class of topological spaces.

4. (a) Show that the absolute value function in R is continuous.

(b) State and prove Lindelof's theorem.

Unit—III

5. (a) Define T_3-space. Prove that a closed subspace of a Normal space is Normal.

(b) Give an example of Hausdorff space. Show that every subspace of T_2-space is a T_2-space.

6. (a) Define regular space. Show that (R, U) is T_3.

(b) Show that every regular Lindelof space is normal.

Unit—IV

7. (a) Give an example of a compact Hausdorff space which is not connected. Prove that every closed subspace of a compact space is compact.

(b) Let (X^*, T^*) be a one point compactification of a non-compact topological space (X, T). Then prove that (X^*, T^*) is Hausdorff if and only if (X, T) is Hausdorff and locally compact.

8. (a) Define sequentially compact with an example. Prove that any open subspace of a locally compact is locally compact.

(b) If A is an infinite subset of a compact space X then prove that A has limit point in X.

Unit—V

9. (a) Give an example of connected set. Prove that the set of real numbers with the usual topology is connected.

   (b) Prove that a topological space X is locally connected if and only if the components of every open subspace of X are open in X.

10. (a) Prove that a metric space is compact if and only if it is complete and totally bounded.

    (b) Give an example of separated set. Let $\{C_\lambda : \lambda \in \Lambda\}$ be a family of connected sets of a topological space having a non-empty intersection. Show that $\cup \{C_\lambda : \lambda \in \Lambda\}$ is connected.

700