Roll No. ....

Z–309

M. A./M. Sc. (First Semester) EXAMINATION, Dec., 2015

MATHEMATICS

Paper First

(Advanced Abstract Algebra—I)

Time : Three Hours] [*Maximum Marks : 80*

[*Minimum Pass Marks : 16*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define p-group and show that if a cyclic group has one and only one composition series, then it is a p-group.

   (b) Define solvable group with example and show that if A and B are two solvable groups, then this product $A \times B$ is also solvable.

   (c) Show that every nilpotent group is solvable.

Unit—II

2. (a) If E is an extension of a filed F then show that F (u) is an algebraic extension of F where $u \in E$ is algebraic over F.

(b) State and prove the theorem of uniqueness of splitting field.

(c) Prove that :

$$Q\left(\sqrt{2}, \sqrt{3}\right) = Q\left(\sqrt{2} + \sqrt{3}\right)$$

Unit—III

3. (a) Show that any *two* finite fields with p^n elements are isomorphic.

(b) Show that $K \cong K'$ where K and K' are algebraic closures of a field F.

(c) Show that the product of two primitive polynomials is also primitive.

Unit—IV

4. (a) Define Galois group and show that the group $G\left(Q(\alpha)/Q\right)$, where $\alpha^5 = 1$ and $\alpha \neq 1$ is isomorphic to the cyclic group of order 4.

(b) Show that *k* is normal extension of F iff G (E/K) is normal subgroup of G (E/F) where E is Galois extension of F and K is any subfield of E containing F.

(c) Find the Galois groups G (K/Q) of the following extensions K of Q :

(i) $K = Q\,(\sqrt{3}, \sqrt{5}$

(ii) $K = Q\,(\alpha)$, where $\alpha = e^{i\frac{2\pi}{3}}$

Unit—V

5. (a) Prove that $f(x) \in F[x]$ is solvable by radicals over F iff its splitting field E over F has solvable Galois group G (E/F).

(b) Test the solvability by radicals over Q for the following polynomials :

(i) $2x^5 - 5x^4 + 5$

(ii) $x^3 - 9x + 3$

(c) Let $f(x) \in Q[x]$ be a monic irreducible polynomial over Q of degree p (prime). If $f(x)$ has exactly two non-real roots in C, then show that the Galois group of $f(x)$ is isomorphic to S_p.

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